Determination of Direct Current Arc Plasma Current Density by Hall-Effect Magnetic Probe

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A simple and novel method of determining the current density of the cylindrical plasma column of an electric arc, based on the measurement of the self-magnetic field due to the arc drift current, is presented. This is accomplished by means of a miniature Hall-effect magnetic probe traversing the column diametrally at approximately 120 cm/sec. The theory of the Hall probe is briefly sketched and the method of data reduction given. The probing unit was tested on the plasma column of the fluid-transpiration arc. Results of several representative probings are presented showing general conformity to theory of the arc self-magnetic field radial profile. The results indicate the fluid-transpiration arc current density to be remarkably uniform over a large cross section of the column. The perturbative effects of the probe insertion on the plasma are analyzed. The advantages of using Hall probes as against coil-type magnetic probes are examined.

Introduction

THE application of the Hall effect for magnetic field measurement is well known.[1] In recent times, the technique has been markedly advanced by modern researches in solid-state physics, where the Hall effect furnishes fundamental insights into charge carrier mobility and concentration. Consequently, Hall-effect devices for measurement of magnetic field, ranging from a fraction of a gauss to tens of kilogausses with an accuracy of $\pm 3\%$ or better, have been made commercially available during the past few years. On the other hand, the use of magnetic probes in plasma research, as revealed in the literature, has, to the best knowledge of this writer, been restricted to coil-type arrangements in which either the discharge current itself must be of a transient nature,[2–5] or of known time variation,[6] or devices in which the coil is required to move relative to the discharge. Both rotary (the usual rotating-coil gaussmeter) and linear motion[7, 8] have been employed.

The primary purpose of this article is to report recent work on the development of a current density probe using the Hall effect, undertaken as part of a diagnostic program of an atmospheric density arc plasma.

Theory

The use of the Hall effect to measure current density in a current-carrying plasma is based on the direct measurement of the self-magnetic field generated by the plasma drift current, followed by appropriate data reduction in accordance with the well-known relation between a current distribution and its associated magnetic field. The present method involves the use of a miniature Hall-effect element to sense the self-magnetic field with good resolution. The relation between the field vectors and the Hall-element output is shown in Fig. 1.

The theory of Hall-effect elements is well covered in the literature.[9, 10] In its simplest form, when all other galvanomagnetic and thermomagnetic effects are negligible, the Hall voltage V_h normal to the direction of current I and magnetic flux density B is

$$V_h = K_h IB/b \tag{1}$$

where b is the dimension of the Hall element in the direction of the magnetic field, and K_h is the Hall coefficient. If $K_h I/b$ is known, then V_h gives an indication of the magnitude and direction of B.

To obtain the current density, one takes, in the limit of vanishing displacement current,

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} \tag{2}$$

and expresses the right-hand side in terms of components of **B** and their spatial derivatives. These components of **B** may be measured by disposing Hall elements in directions appropriate to the coordinate system chosen.

In a cylindrical coordinate system,

$$\mathbf{J} = \mu_0^{-1}\nabla \times \mathbf{B} = \mu_0^{-1}\{J_r, J_\theta, J_z\} = \mu_0^{-1}\left\{\frac{1}{r}\frac{\partial B_z}{\partial \theta} - \frac{\partial B_\theta}{\partial z}, \frac{\partial B_r}{\partial z} - \frac{\partial B_z}{\partial r}, \frac{1}{r}\frac{\partial}{\partial r}(rB_\theta) - \frac{1}{r}\frac{\partial B_r}{\partial \theta}\right\} \tag{3}$$

Thus, when there is rotational symmetry in **J**,

$$\mathbf{J} = \{0, 0, J_z\} = \mu_0^{-1}\{0, 0, (\partial B_\theta/\partial r) + (B_\theta/r)\} \tag{4}$$

Thus, only one Hall element is necessary to determine the current density.

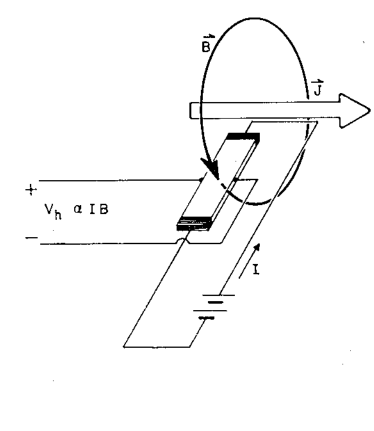

Fig. 1 Relation between field vectors and Hall voltage output.

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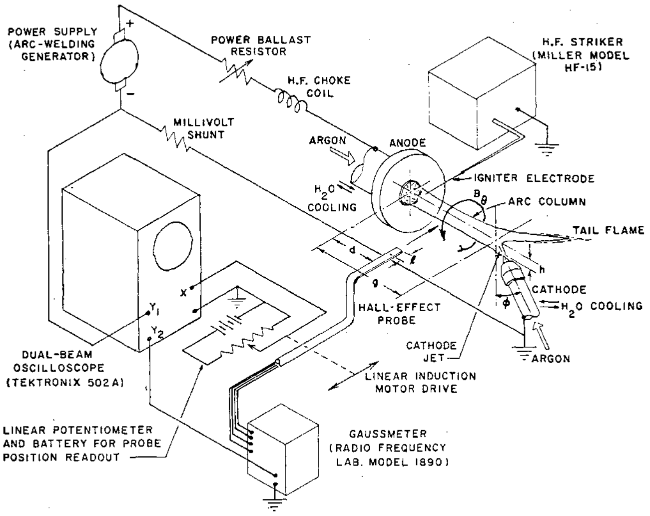

Fig. 2 Experimental arrangement of fluid transpiration arc, Hall probe, and associated instrumentation.

Experimental Arrangement

Figure 2 shows the arrangement of arc equipment, Hall probe, and the associated instrumentation. The type of plasma generator used in this study is reported by Sheer, Cooney, and Rothacker.[11] In this device, the working fluid (argon) is passed through a porous anode (National Carbon Company NC-50 grade) machined into a disk 0.32 cm thick and having a diameter, on the arc side, of 1.1 cm. The cathode consists of a conically tipped $\frac{1}{4}$-in. thoriated tungsten rod and is protected from the atmosphere by a separate argon gas mantle. The main stream of argon is introduced into the arc column through the porous anode. The arc is initiated by high-frequency discharge, and the d.c. arc is sustained by an arc-welding generator with a current fluctuation of approximately 5% of total.

To obtain the self-magnetic field of the arc column, it is necessary that the probe be inserted into the conduction zone, which in this case may reach a temperature well in excess of 5000°K. For this reason, the immersion time must be as short as possible. Also, because most Hall probe materials (materials with reasonably high Hall coefficients) have low melting points, and, of greater importance, high temperature coefficients, thermal shielding of the probe is mandatory even for very short residence times. We used asbestos paper shaped into a scabbard, which is tapered to hug the contours of an InAs Hall probe (Radio Frequency Laboratory HB-13475, 0.020 in. thick × $\frac{1}{8}$ in. wide × $\frac{9}{32}$ in. long) and coated on the outside with a thin layer of silicone rubber.

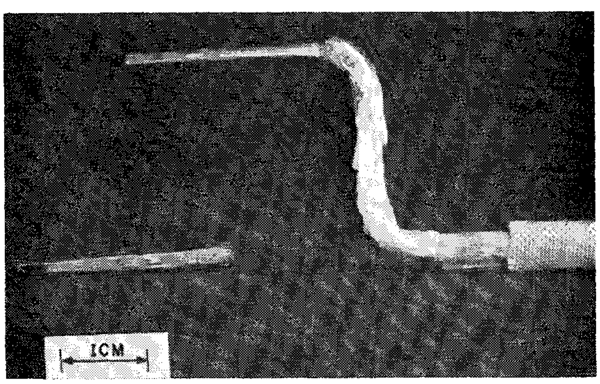

Fig. 3 InAs Hall probe showing the exposed sensing element and the protective sheath. (This is not a true side view; the actual thickness of the sensing element is 0.020 in.).

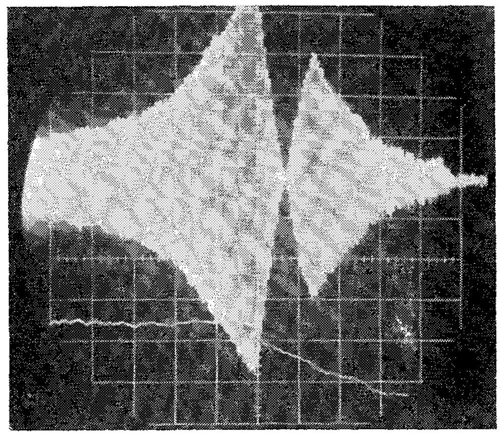

Fig. 4 Hall probe output and total arc current vs radial traverse (left to right) 3.38 gauss/major div. (peak-to-peak); 50 amp/major div.; 0.565 cm/major div.; d = 0.2 cm, g = 2.3 cm, $h \approx$ 0.5 cm, anode flow = 23 g/min, cathode flow = 5 g/min, arc voltage = 40 v.

The finished assembly is shown with the shield detached in Fig. 3. Notice that the probe stem is bent into the general shape of a step in order to facilitate single traverses through the axis of the arc without burning up the probe stem and the leads inside. The Radio Frequency Laboratory Model 1890 Gaussmeter has a 3000 cps current generator feeding the Hall element; hence, the Hall voltage is in the form of a 3000 cps carrier signal, amplitude-modulated by the local magnetic field. The output is filtered and amplified before being displayed by one beam (Y_2) of a dual-beam oscilloscope. The other beam (Y_1) gives a display of the total arc current by means of a 50 mv shunt.

The radial position (with respect to the arc column axis) of the probe is monitored by a precision 20,000-ohm potentiometer with a 9-v dry cell, and the output is connected to the horizontal sweep (X) of the oscilloscope. The traces are recorded photographically on an oscilloscope camera. The entire probe assembly is transported by a linear induction motor-drive mechanism having a reversible stroke of 4 in. and a speed at midstroke of approximately 120 cm/sec.

Results

Typically, the arc is maintained with a gap (g in Fig. 2) of 2 to 3 cm and a current of 125 amp and the column terminates on about 90% of the anode face. There is usually a slight reduction of arc column radius toward the cathode, based on luminosity observation. The cathode position and the argon flows are adjusted to minimize instability as well as to limit the extent of the cathode jet; the latter interferes with the achievement of over-all rotational symmetry. Therefore a large gap was used, and measurements were restricted initially to portions of the column proximal to the anode face.

Figure 4 shows a representative picture of the Gaussmeter output and total arc current as a function of radial traverse (starting at the left) diametrally through the arc column. The magnetic field is seen to increase as the probe approaches the cylindrical column boundary, reaches a maximum, and then decreases once inside the column. It then goes through a null and increases again (actually with an opposite sign, not discernible in the carrier envelope) until a second maximum is reached corresponding to the opposite column boundary. Then it decreases as the probe departs from the column. It will be noted in Fig. 4, that the second peak is lower by about 40% than the first peak. This reflects a reduction of the arc column current (and therefore the self-magnetic field) by about the same amount, as indicated by the downward slope of the arc current trace in the lower part of the picture. The dropping off of the plasma current as the probe advances into the column is assumed to be due to the obstructing effect of the probe.

Figures 5 and 6 were obtained under identical conditions with the exception that, in the case of Fig. 6, an extension of 1 cm was butt-joined to the leading edge (looking toward the arc column) of the old sheath.

Discussion

Insertion of probes into current-carrying plasmas perturbs the conduction column to a greater or lesser extent, depending on the nature of the plasma being probed and the size of the probe relative to the conduction zone. In atmospheric-pressure arcs, the contracted cathode arc stream† is generally more sensitive than the positive column. The plasma emerging from the porous anode of the fluid transpiration arc appears to be particularly insensitive to probing. At least there were no visual indications of deflection or distortion of the column, as is often observed when probes of conducting material or of non-negligible dimensions are inserted into high-pressure gas discharges.

In the present work, the probe thickness, including heat shield, varied from 0.15 to 0.25 cm. Since the plasma column was only about 1 cm in diameter, the probe occupied an appreciable fraction of the column dimensions. Nevertheless, the only perturbing effect of probe insertion observed was a reduction of the plasma current in the approximate proportion of the obstructing probe to the conduction zone cross section. In this respect, it is important to note that all the recorded traces showed a null point at the column axis.‡ This, of course, indicates that the sensitive area of the Hall element was small enough to give excellent spatial resolution.

When the left-hand portion of Fig. 4 was plotted vs distance on a log-log paper, the field is seen to vary as r^{-1} up to 0.45 cm from the apparent column edge. At this point it starts to fall below the theoretical value. At the column edge the probe output is 11.5 gauss below theoretical. The reason for this may be clarified by referring to Fig. 7a, which illustrates the effect of the Hall-element sensing spot being well behind the shield leading edge. The exclusion of the original current from this zone significantly reduces the probe reading, because the obstructed portion of the original current, according to Biot-Savart Law, would have made a large contribution to the field at the sensing spot by reason of proximity.§ When the probe advances further, as shown in Fig. 7b, the effects of the obstructed currents on both sides of the sensing spot tend to cancel each other out, and the probe output more closely approximates the original field. Figure 7c shows a large area of the original column being obstructed by the probe holder subsequent to the Hall-element traverse. As mentioned earlier, large perturbations result even to the point of quenching the arc. Usually, however, this merely makes the probe output unsymmetrical. Figures 5 and 6 further confirm the effect of the leading edge, since here the onset of departure from theoretical values (before the sensing element reaches the conduction zone) is seen to be advanced by an amount given by the length of the sheath extension. Also, note the same locations for the null points (arc axis) for both runs.

The width of the arc column obtained from the probe results agrees quite well with visual observation.

† The "negative column" or portion of the discharge near the cathode.

‡ Strictly speaking, a null point will show whenever the probe plane is tangential to the field lines. Since the probe is not far off the diametral path, the null indicates closely the arc axis.

§ By approximating the effect of this obstructed current element by an equivalent column, correction of the order of 15 gauss is easily obtained. Also, the off symmetry of the cathode can be taken into account.

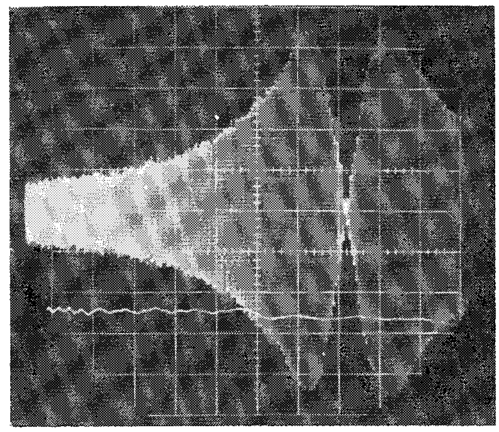

Fig. 5 Same as in Fig. 4 except g = 2.4 cm, arc voltage = 45 v.

Current Density of the Fluid-Transpiration Arc

Assuming rotational symmetry, the current density J_z is given by

$$J_z = \mu_0^{-1}(\partial B_\theta/\partial r) + (B_\theta/r) \qquad 0 < r < r_{\text{column}} \tag{5}$$

In most cases studied so far, B_θ is given with a good degree of accuracy by the expression

$$B_\theta \doteq Ar \tag{6}$$

where A is a constant. Equation (5) is thereby simplified to

$$J_z \doteq 2\mu_0^{-1} A \doteq \text{const} \tag{7}$$

For example, the current density obtained from Fig. 5 is 142 amp/cm² as compared with 173 amp/cm² based on measured total current (a rather reasonable agreement, considering the possible departures from the idealized case).

Referring again to the trace of Fig. 5, it will be observed that the envelope of the Hall probe output is quite linear within the column from the axis almost to the column boundary on each side.¶ This of course confirms Eq. (6) and, by Eq. (7), means that the current density is uniform over nearly the entire cross section of the plasma column. Such a flat-topped radial distribution of current implies a correspondingly uniform radial temperature distribution which would be a rare and valuable feature for an unconfined plasma column. This feature is presently being checked by temperature measurement.

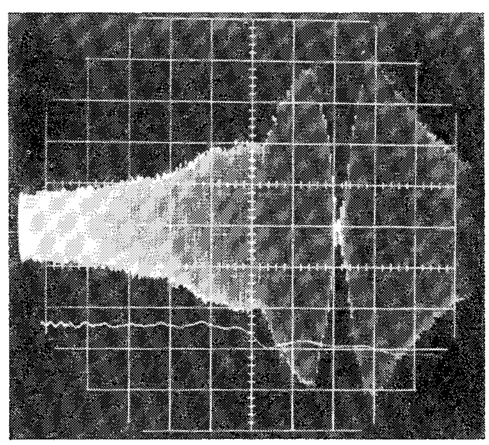

Fig. 6 Same as in Fig. 5 except for a 1-cm extension of the probe sheath leading edge.

¶ This has been checked by photographic enlargements as well as by magnifying the horizontal sweep rate.

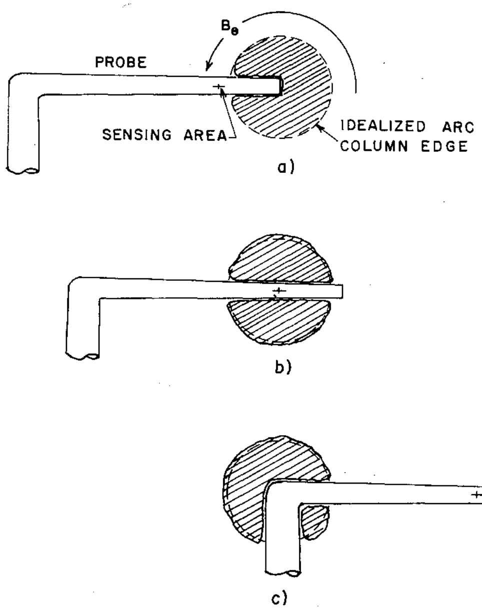

Fig. 7 Idealized sketches of the stages of probing.

Advantages of the Hall-Element Probe for Plasma Diagnostics

There are several advantages that the Hall probe offers in comparison with the other methods. These are as follows.

1. High Sensitivity and Wide Range

Any coil technique of magnetic field measurement depends on time rate of change of enclosed flux. This is relatively simple to measure when either high-speed transient or very high current density is involved. But, in d.c. or slowly varying discharges operating between a few amperes and several hundreds of amperes, the requirements for high flux-cutting speed and large numbers of turns take on very challenging aspects if output of useful magnitude is to be had. The Hall probe is especially suited to this low range as well as to the higher ranges.

2. Small Size

A smaller probe is naturally to be desired over a larger one having the same sensitivity. In contrast to a multiturn coil, whose output is proportional to the loop area, Hall probe measures true flux density subject only to fringe effect considerations and is therefore more easily miniaturized. Roshon[12] has reported a microprobe having a $10 \times 10\ \mu$ sensitive area, using an evaporated bismuth film. Thus, small probes having three orthogonally positioned Hall elements may be used to detect arbitrary fields without appreciable perturbation of the plasma.

3. Mechanical Simplicity

No precise control of probe movement is necessary, and no commutation problem arises, as in rotating-coil gaussmeter.

The chief limitation of the Hall probe is the temperature variation of the Hall coefficient. However, this has presented no serious obstacle with a moderately fast traversing mechanism and conventional thermal insulation in the hostile environment of an arc plasma. That an appropriately designed probe may survive other types of plasma media is perhaps not unreasonable. The high-frequency response, important in pulse discharges, is limited by the mean collision time of the Hall-element conduction electrons and by the surface (skin) Hall effects, but it appears that a few kilomegacycles can be attained.

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